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Three-Mode Entangled State Representation of Continuum Variables and Optical Four-Wave Mixing

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We establish a new three-mode entangled state representation $|\beta, \zeta\rangle_\theta$ of continuum variables, which make up a complete set. Using optical four-wave mixing and a beam splitter transform we can prepare $|\beta, \zeta\rangle_\theta$. Based on $|\beta, \zeta\rangle_\theta$ a new number-difference—operational-phase uncertainty relation is established and the corresponding squeezing dynamics is discussed.

KEY WORDS: three-mode entangled state representation; beam splitter transform; four-wave mixing process.

1. INTRODUCTION

It is well known that an optical four-wave mixing process (Scully and Zubairy, 2000; Loudon and Knight, 1987) is a nonlinear one in which two planar counter-propagating intense pump waves ε_1 and ε_1' interact in a nonlinear medium (characterized by a third-order nonlinear susceptibility $\chi^{(3)}$, the length of medium is L) with a probe field ε_2 entering at an arbitrary angle to the pump waves and yield a fourth (output) wave ε_3, which is the phase conjugate of ε_2. The generated field is driven only by the complex conjugate of the signal field amplitude, thus leading to phase conjugate. By extending the classical analysis to phenomenological quantum treatment (the fields are specified at the ends of the nonlinear crystal, $z = 0$ and L) one can derive that the signal and its conjugate fields (weaker than pump fields) satisfy the equations (Scully and Zubairy, 2000)

$$\frac{da_2}{dz} = i\kappa a_3^\dagger, \qquad \frac{da_3}{dz} = i\kappa a_2^\dagger, \tag{1}$$

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where κ is a parameter relating to the third-order nonlinear susceptibility $\chi^{(3)}$. Let $a_2(0)$ of the probe field ε_2 and $a_3(L)$ of ε_3 be the initial known fields, then the solution of (1) is (Scully and Zubairy, 2000) (see the appendix).

$$a_2(L) = a_2(0)\sec\theta + i\frac{\kappa}{|\kappa|}a_3^\dagger(L)\tan\theta, \quad \theta = |\kappa|L,$$
$$a_3(0) = a_3(L)\sec\theta + i\frac{\kappa}{|\kappa|}a_2^\dagger(0)\tan\theta. \tag{2}$$

These solutions, more or less, resemble the two-mode squeezing transform, however, its form somehow differs from the well-known Bogolyubov transform (the usual two-mode squeezing transform is $a_2 \rightarrow a_2\cosh\tau + a_3^\dagger\sinh\tau,\ a_3 \rightarrow a_3\cosh\tau + a_2^\dagger\sinh\tau$, which happens in a parametric down-conversion process (Loudon and Knight, 1987)) so we name (2) the four-wave mixing transform. On the other hand, in recent years quantum entanglement and entangled states have been paid much attention in quantum optics. For example, Tara and Agarwal (1994) discussed how the Einstein–Podolsky–Rosen paradox for continuous variables can be tested using the quadrature amplitudes of a radiation field in the pair-coherent state. Correlated pairs of photons are produced by two competing nonlinear processes—four-wave mixing and two-photon absorption. Fan and Klauder (1994) (Fan *et al.*, 2003) constructed in Fock space the bipartite entangled state $|\beta\rangle = \exp[-\frac{1}{2}|\beta|^2 + \beta a_1^\dagger + \beta^* a_2^\dagger - a_2^\dagger a_1^\dagger]\,|00\rangle$ and later it is shown that the two-mode squeezing operator $\exp[\lambda(a_1^\dagger a_2^\dagger - a_1 a_2)]$ just squeezes $|\beta\rangle \rightarrow |\beta/\mu\rangle/\mu,\ \mu = e^\lambda$ (Fan and Fan, 1996). It is also shown that the operational-phase operator proposed by Noh *et al.* (1991) and analyzed deeply by Freyberger *et al.* (1995) manifestly exhibits its phase behavior in the bipartite entangled state representation (Fan and Min, 1996). All these indicate that establishing suitable entangled state of continuum variables will be of help in many situations. An interesting question thus naturally arises: Can we employ the four-wave mixing mechanism to construct a type of three-mode entangled states? The answer is affirmative. In Section 2 we present such an entangled state and then discuss its generation. In Section 3 we discuss its major properties. In Section 4 we show that the new three-mode entangled state can lead us to construct a new number-difference—operational-phase uncertainty relation in three-mode case. In Section 5 we discuss the dynamics governing the number-difference—operational-phase squeezing.

2. THREE-MODE ENTANGLED STATE $|\beta, \zeta\rangle_\theta$

We construct the following three-mode entangled states

$$|\beta,\zeta\rangle_\theta = \cos\theta\exp\Big[-\frac{1}{2}\,(|\beta|^2 + |\zeta|^2) + \beta a_1^\dagger + \zeta a_3^\dagger + a_2^\dagger(a_1^\dagger - \beta^*)\cos\theta - i a_2^\dagger(a_3^\dagger - \zeta^*)\sin\theta\Big]|000\rangle. \tag{3}$$

where β, ζ are two complex variables. This type of state is comparatively simpler in form. In order to see how four-wave mixing mechanism can engender it, we introduce an operator S that induces the four-wave mixing transform like (2),

$$a_2^\dagger \to S a_2^\dagger S^{-1} = a_2^\dagger \sec\theta - i a_3 \tan\theta, \quad a_3^\dagger \to S a_3^\dagger S^{-1} = a_3^\dagger \sec\theta - i a_2 \tan\theta. \tag{4}$$

We can derive the concrete form of S by a mapping from the classical transform $\alpha_2 \to \alpha_2 \sec\theta - i\alpha_3^* \tan\theta$, $\alpha_3 \to -i\alpha_2^* \tan\theta + \alpha_3 \sec\theta$ in the coherent state basis to Hilbert space, i.e.

$$S = \int \frac{d^2\alpha_3 d^2\alpha_2}{\pi^2} |\alpha_2 \sec\theta - i\alpha_3^* \tan\theta, \quad i\alpha_2^* \tan\theta + \alpha_3 \sec\theta\rangle \langle \alpha_2, \alpha_3|, \tag{5}$$

where $\langle \alpha_2, \alpha_3|$ is the two-mode coherent state (Klauder and Skargerstam, 1985; Glauber, 1963)

$$\langle \alpha_2, \alpha_3| = \langle 0, 0| \exp\left[-\frac{1}{2}|\alpha_2|^2 + |\alpha_3|^2 + a_2\alpha_2^* + a_3\alpha_3^*\right]. \tag{6}$$

Using the normal ordering form of vacuum projector

$$|0, 0\rangle \langle 0, 0| =: \exp[-a_2^\dagger a_2 - a_3^\dagger a_3] :, \tag{7}$$

where the symbol : : denotes normal ordering, and the technique of integration within an ordered product (IWOP) of operators (Fan, 2003; Wunshe, 1999) we perform the integral in (5) and obtain

$$\begin{aligned} S = \int \frac{d^2\alpha_3 d^2\alpha_2}{\pi^2} : \Bigg[& -\frac{1}{2}|\alpha_2 \sec\theta - i\alpha_3^* \tan\theta|^2 + (\alpha_2 \sec\theta - i\alpha_3^* \tan\theta)\, a_2^\dagger \\ & -\frac{1}{2}|-i\alpha_2^* \tan\theta + \alpha_3 \sec\theta|^2 + (-i\alpha_2^* \tan\theta + \alpha_3 \sec\theta)\, a_3^\dagger \\ & -\frac{1}{2}(|\alpha_2|^2 + |\alpha_3|^2) + a_2\alpha_2^* + \alpha_3^* a_3 - a_2^\dagger a_2 - a_3^\dagger a_3 \Bigg] : \\ = \cos\theta \exp[-i a_3^\dagger a_2^\dagger \sin\theta] & \exp[(a_2^\dagger a_2 + a_3^\dagger a_3)\, \text{In} \cos\theta] \exp[-i a_3 a_2 \sin\theta]. \end{aligned} \tag{8}$$

Using Baker–Hausdorff formula we confirm that (8) really induces (4).

It has been reported (Loock and Braunstein, 2000; Cochrane and Milburn, 2001) that an ideal beam splitter's operation applied to a momentum squeezed (maximally squeezed in the P-quadrature direction) vacuum state (mode $a_1^\dagger$) and a position-squeezed (maximally squeezed in the X-quadrature direction) vacuum state (mode $a_2^\dagger$) can yield a two-mode entangled state $\exp[a_1^\dagger a_2^\dagger]\, |00\rangle_{12}$. On the basis of this state, we let $a_2^\dagger$-mode, the probe field ε_2, be coupled with another mode $a_3^\dagger$ in the four-wave mixing equipment then the result can be theoretically expressed by operating S on the state $\exp[a_1^\dagger a_2^\dagger]\,|00\rangle_{12} \otimes |0\rangle_3$, i.e.,

$$\begin{aligned} S\exp[a_1^\dagger a_2^\dagger]|000\rangle &= \exp[a_1^\dagger\,(a_2^\dagger \sec\theta - ia_3\tan\theta)]S|000\rangle \\ &= \cos\theta\exp[a_1^\dagger(a_2^\dagger\sec\theta - ia_3\tan\theta)]\exp[-a_2^\dagger a_3^\dagger\sin\theta]|000\rangle \\ &= \cos\theta\exp[a_1^\dagger a_2^\dagger\cos\theta - ia_3^\dagger a_2^\dagger\sin\theta]|000\rangle = |\beta = 0, \zeta = 0\rangle_\theta. \end{aligned} \tag{9}$$

By further operating the two-mode displacement operator $D_1(\beta)D_3(\zeta)$ on $|\beta = 0, \zeta = 0\rangle_\theta$, where $D_1(\beta) = \exp(\beta a_1^\dagger - \beta^* a_1)$, $D_3(\zeta) = \exp(\zeta a_3^\dagger - \zeta^* a_3)$ (this displacement can be realized by two local oscillators), we have

$$D_1(\beta)\,D_3(\zeta)\,|\beta = 0, \zeta = 0\rangle_\theta = |\beta, \zeta\rangle_\theta\,, \tag{10}$$

which is just (3).

3. PROPERTIES OF $|\beta, \zeta\rangle_\theta$

It stands to reason that $|\beta, \zeta\rangle_\theta$ is a generalized entangled state, in which $a_2^\dagger$ mode entangles both $a_3^\dagger$ and $a_1^\dagger$ modes. When $\theta = \pi$, $|\beta, \zeta\rangle_\theta \to |\beta\rangle_{12} \otimes |\zeta\rangle_3$, where

$$|\beta\rangle_{12} = \exp\left[-\frac{1}{2}\,|\beta|^2 + \beta a_1^\dagger + \beta^* a_2^\dagger - a_2^\dagger a_1^\dagger\right]|00\rangle_{12}, \tag{11}$$

is the two-mode entangled state in modes 2 and 3, and

$$|\zeta\rangle_3 = \exp\left[-\frac{1}{2}\,|\zeta|^2 + \zeta a_3^\dagger\right]|0\rangle_3. \tag{12}$$

is a coherent state in mode 3. Thus we see that when one makes a correlation between single-mode coherent states with a two-mode entangled state, one can obtain a non-trivial three-mode entangled state.

From (3) we obtain three independent eigenvector equations,

$$(a_1 - a_2^\dagger\cos\theta)\,|\beta, \zeta\rangle_\theta = \beta\,|\beta, \zeta\rangle_\theta\,, \tag{13}$$

$$(a_3 - ia_2^\dagger\sin\theta)\,|\beta, \zeta\rangle_\theta = \zeta\,|\beta, \zeta\rangle_\theta\,, \tag{14}$$

$$(a_2 - a_1^\dagger\cos\theta + ia_3^\dagger\sin\theta)\,|\beta, \zeta\rangle_\theta = (-\beta^*\cos\theta + i\zeta^*\sin\theta)\,|\beta, \zeta\rangle_\theta\,, \tag{15}$$

Combining (13) and (14), we have

$$(a_2^\dagger - a_1\cos\theta - ia_3\sin\theta)\,|\beta, \zeta\rangle_\theta = (-\beta\cos\theta - i\zeta\sin\theta)\,|\beta, \zeta\rangle_\theta\,. \tag{16}$$

and

$$(a_3\cos\theta + ia_1\sin\theta)\,|\beta, \zeta\rangle_\theta = (\zeta\cos\theta + i\beta\sin\theta)\,|\beta, \zeta\rangle_\theta\,. \tag{17}$$

By introducing $X_3 = \frac{1}{\sqrt{2}}(a_3 + a_3^\dagger)$, $P_3 = \frac{1}{\sqrt{2}i}(a_3 - a_3^\dagger)$, (15) and (16) are equivalent to

$$(X_2 - X_1 \cos\theta + P_3 \sin\theta)\,|\beta,\zeta\rangle_\theta = \sqrt{2}\,(\zeta_2 \sin\theta - \beta_1 \cos\theta)\,|\beta,\zeta\rangle_\theta\,, \tag{18}$$

$$(P_2 + P_1 \cos\theta + X_3 \sin\theta)\,|\beta,\zeta\rangle_\theta = \sqrt{2}\,(\beta_2 \cos\theta + \zeta_1 \sin\theta)\,|\beta,\zeta\rangle_\theta\,. \tag{19}$$

Note that the three operators $(X_2 - X_1 \cos\theta + P_3 \sin\theta)$, $(P_2 + P_1 \cos\theta + X_3 \sin\theta)$ and $(a_3 \cos\theta + i a_1 \sin\theta)$ constitute a complete commutable operator set. Using the technique of IWOP, we can concisely prove

$$\int \frac{d^2\beta d^2\zeta}{\pi^2 \cos^2\theta}\,|\beta,\zeta\rangle_{\theta\theta}\,\langle\beta,\zeta| =: \exp\left[a_2^\dagger a_2(\sin^2\theta + \cos^2\theta - 1)\right] := 1. \tag{20}$$

This is the completeness relation of $|\beta,\zeta\rangle_\theta$. We also calculate

$$\begin{aligned}{}_\theta\left\langle \beta',\zeta'\,|\beta,\zeta\right\rangle_\theta &= \pi\delta[(\beta_2 - \beta_2') + (\zeta_1 - \zeta_1')\tan\theta]\delta[(\beta_1 - \beta_1') - (\zeta_2 - \zeta_2')\tan\theta] \\ &\quad \times \exp\left\{\beta'^*\beta + \zeta'^*\zeta - \frac{1}{2}\left(|\beta|^2 + |\zeta|^2 + |\beta'|^2 + |\zeta'|^2\right)\right\},\end{aligned} \tag{21}$$

which shows that the state $|\beta,\zeta\rangle_\theta$ is partly orthogonal.

Thus the ideal $|\beta,\zeta\rangle_\theta$ is of importance not only because it can be produced experimentally, but also because it is qualified to make up a new quantum mechanical representation.

4. NEW NUMBER-DIFFERENCE—OPERATIONAL-PHASE UNCERTAINTY RELATION IN THREE-MODE CASE

Based on $|\beta,\zeta\rangle_\theta$ we can construct a new number-difference—operational-phase uncertainty relation. In quantum optics theory, number-phase uncertainty relation has been a hot topic for many years. In (Noh *et al.*, 1991, 1993) based on the eight-port homodyne detection scheme Noh, Fougeres and Mandel have introduced the two-mode operational-phase operator $\sqrt{(a_1 - a_2^\dagger)/(a_1^\dagger - a_2)}$ (note $[a_1 - a_2^\dagger, a_1^\dagger - a_2] = 0$, so they can reside in the same square root), which can be diagonalized in the bipartite entangled state representation (Fan and Min, 1996). If the $a_2^\dagger$-mode undergoes a four-wave mixing transform before it enters into an eight-port homodyne interferometer, then according to (3) the operational-phase operator should be modified as

$$\begin{aligned}\sqrt{\frac{a_1 - a_2^\dagger}{a_1^\dagger - a_2}} \to \widehat{e^{i\Phi}} &= \sqrt{\frac{a_1 - a_2^\dagger \sec\theta + i a_3 \tan\theta}{a_1^\dagger - a_2 \sec\theta - i a_3^\dagger \tan\theta}} \\ &= \sqrt{\frac{a_2^\dagger - i a_3 \sin\theta - a_1 \cos\theta}{a_2 + i a_3^\dagger \sin\theta - a_1^\dagger \cos\theta}}.\end{aligned} \tag{22}$$

$\widehat{e^{i\Phi}}$ is unitary. Using (15), (16) and (20) we see that $\widehat{e^{i\Phi}}$ is diagonalized in $|\beta, \zeta\rangle_\theta$ representation,

$$\begin{aligned}\widehat{e^{i\Phi}} &= \int \frac{d^2\beta d^2\zeta}{\pi^2 \cos^2\theta} \left(\frac{-\beta\cos\theta - i\zeta\sin\theta}{-\beta^*\cos\theta - i\zeta^*\sin\theta}\right)^{1/2} |\beta, \zeta\rangle_{\theta\theta}\langle\beta, \zeta| \\ &= \int \frac{d^2\beta d^2\zeta}{\pi^2 \cos^2\theta} \exp[i \arg(-\beta\cos\theta - i\zeta\sin\theta)]|\beta, \zeta\rangle_{\theta\theta}\langle\beta, \zeta|.\end{aligned} \tag{23}$$

Let

$$A = a_2 + ia_3^\dagger \sin\theta - a_1^\dagger \cos\theta, \tag{24}$$

We can make the polar decomposition

$$A = \widehat{e^{-i\Phi}}\sqrt{A^\dagger A,} \qquad A^\dagger = \sqrt{A^\dagger A}\widehat{e^{i\Phi}}, \tag{25}$$

where

$$A^\dagger A = \frac{1}{2}[(X_2 - X_1\cos\theta + P_3\sin\theta)^2 + (P_2 + P_1\cos\theta + X_3\sin\theta)^2]. \tag{26}$$

After many trials we find that the variable D which is conjugate to the phase angle $\hat{\Phi}$,

$$D = a_1^\dagger a_1 - a_2^\dagger a_2 + a_3^\dagger a_3, \tag{27}$$

such an operator may describe optical radiation from two energy levels and simultaneously absorbed by another energy level. We can prove

$$[D, A^\dagger] = -A^\dagger, \qquad [D, A] = A, \qquad [D, A^\dagger A] = 0. \tag{28}$$

The commutative relation between D and the phase operator is

$$[D, \widehat{e^{i\Phi}}] = -\widehat{e^{i\Phi}}, \quad [\widehat{e^{-i\Phi}}, D] = -\widehat{e^{-i\Phi}}, \quad [\hat{\Phi}, D] = -i. \tag{29}$$

It then follows a new generalized number-difference—operational-phase uncertainty relation

$$\Delta D\, \Delta\cos\hat{\Phi}\frac{1}{2}|\sin\hat{\Phi}|. \tag{30}$$

5. NUMBER-DIFFERENCE—OPERATIONAL-PHASE SQUEEZING DYNAMICS

Based on (29) we can also discuss the number-difference—operational-phase squeezing dynamics in three-mode case. Spirit in similar to (Collet, 1993), we introduce the following Hamiltonian in the interaction picture

$$H_1 = \frac{1}{2}g(D\sin\hat{\Phi} + \sin\hat{\Phi}D), \tag{31}$$

where g is the coupling constant. From the Heisenberg equation we have

$$\frac{dD}{dt} = -i\,[D, H_I] = \frac{g}{2}\{D, \cos\hat{\Phi}\}, \tag{32}$$

where $\{A, B\}$ denotes $AB + BA$, and

$$\frac{d}{dt}\sin\hat{\Phi} = -\frac{g}{2}\sin 2\hat{\Phi}, \qquad \frac{d}{dt}\cos\hat{\Phi} = g\sin^2\hat{\Phi}. \tag{33}$$

It then follows

$$\frac{d}{dt}\tan\frac{1}{2}\hat{\Phi} = \frac{d}{dt}\left(\frac{1-\cos\hat{\Phi}}{\sin\hat{\Phi}}\right) = -g\tan\frac{1}{2}\hat{\Phi}. \tag{34}$$

The solution to (34) is

$$\tan\frac{1}{2}\hat{\Phi} = e^{-gt}\tan\frac{1}{2}\hat{\Phi}(0), \tag{35}$$

the factor e^{-gt} explains the time evolution, so H_I is the Hamiltonian for phase squeezing in three-mode case. On the other hand, from (29) we have

$$[D, \{D, \sin\hat{\Phi}\}] = i\{D, \cos\hat{\Phi}\}, \qquad [D, -\{D, \cos\hat{\Phi}\}] = i\{D, \sin\hat{\Phi}\}. \tag{36}$$

Noting that

$$\sin\hat{\Phi}D^2\cos\hat{\Phi} = \frac{1}{2}D^2\sin 2\hat{\Phi} - i\{\cos\hat{\Phi}, D\}\cos\hat{\Phi}, \tag{37}$$

$$\cos\hat{\Phi}D^2\sin\hat{\Phi} = \frac{1}{2}D^2\sin 2\hat{\Phi} + i\{\sin\hat{\Phi}, D\}\sin\hat{\Phi}, \tag{38}$$

$$\sin\hat{\Phi}D\sin\hat{\Phi} + \cos\hat{\Phi}D\cos\hat{\Phi} = D, \tag{39}$$

$$[\sin\hat{\Phi}D, \cos\hat{\Phi}D] = [D\sin\hat{\Phi}, D\cos\hat{\Phi}] = -iD, \tag{40}$$

we can derive

$$[\{D, \cos\hat{\Phi}\}, \{D, \sin\hat{\Phi}\}] = 4iD. \tag{41}$$

From (36–41) we see

$$D, \frac{1}{2}i\{D, \cos\hat{\Phi}\}, \quad -\frac{1}{2}i\{D, \sin\hat{\Phi}\}, \tag{42}$$

constitute a close SU(2) Li algebra. So the time evolution of operator D is

$$D(t) = e^{iH_I t}D(0)e^{-iH_I t} = \frac{1}{2}(e^{gt}J_+ + e^{-gt}J_-). \tag{43}$$

where

$$J_+ = \frac{1}{2}\{D(0), 1-\cos\hat{\Phi}\},\ J_- = \frac{1}{2}\{D(0), 1+\cos\hat{\Phi}\}. \tag{44}$$

In summary, we have shown how a four-wave mixing transform, together with a beam splitter transform, can engender the three-mode entangled state (3). By combining a beam splitter and a four-wave mixing setup to make up a triter device in which the two processes coexist, we can engender the new three-mode entangled state $|\beta, \zeta\rangle_\theta$. Based on $|\beta, \zeta\rangle_\theta$ a new number-difference—operational-phase uncertainty relation for three modes is established and the Hamiltonian for number-difference—operational-phase squeezing in three-mode case can be derived.

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APPENDIX

The solution of (1) can be

$$a_2 = C_1 \sin|\kappa|z + C_2 \cos|\kappa|z, \qquad a_3 = D_1 \sin|\kappa|z + D_2 \cos|\kappa|z. \tag{45}$$

Substituting (45) into (1) yields

$$\begin{aligned} |\kappa|\,(C_1 \cos|\kappa|\,z - C_2 \sin|\kappa|\,z) &= ika_3^\dagger, \\ |\kappa|\,(D_1 \cos|\kappa|\,z - D_2 \sin|\kappa|\,z) &= ika_2^\dagger, \end{aligned} \tag{46}$$

where C_i and D_i $(i = 1, 2)$ are integral constants to be determined. Let $a_2(0)$ of the probe field ε_2 and $a_3(L)$ of ε_3 be the initial known fields, then from (45) and (46) one obtains

$$\begin{aligned} C_2 &= a_2(0), \\ D_1 \sin|\kappa|L + D_2 \cos|\kappa|L &= a_3(L), \\ |\kappa|(C_1 \cos|\kappa|L - C_2 \sin|\kappa|L) &= ika_3^\dagger(L), \\ |\kappa|D_1 &= -ika_2^\dagger(0). \end{aligned} \tag{47}$$

After some simple algebra working out the explicit form of C_2 and D_2 and then substituting them into (45) one can reach result (2).

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